Reprinted from Water Forum '86: World Water Issues in Evolution
Proceedings of ASCE Conference/HY, IR, EE, WR, WW Divs.
Long Beach, CA, August 4-6, 1986

Parameter Optimization of Dynamic Routing Models

D.L. Fread and J.M. Lewis*

Abstract - A methodology has been developed for determining the optimal parameters of dynamic routing models thereby eliminating costly and time-consuming preparation of detailed cross-sectional data. The methodology utilizes (1) approximate cross-sectional properties represented by separate power functions for channel and floodplain, and (2) a very efficient optimization algorithm for determining the Manning n as a function of either stage or discharge. Essential data required for implementing the methodology are stage hydrographs at both ends of each routing reach and a discharge hydrograph at the upstream end of each river. The methodology is applicable to multiple routing reaches along main-stem rivers and their tributaries. Optimal n values may be constrained to fall within a specified min-max range for each routing reach. Specific cross-sectional properties at key locations, e.g., bridges, dams, unusual constrictions, also can be utilized within the optimization methodology. The methodology was tested on 1275 miles (2051 km) of major rivers and their principal tributaries in the U.S. with promising results; the average root-mean-square error was 0.44 ft (0.13 m) or 2.9 percent of the change in stage.

Introduction

Flood routing is an essential tool for flood forecasting and engineering design or analysis of hydraulic structures. Of the many flood routing methods that have been developed, dynamic flood routing based on a four-point implicit finite difference solution of the one-dimensional equations of unsteady flow (Saint-Venant equations) has generally been accepted as the most powerful with feasible computational requirements. Dynamic routing enables the prediction of water elevations (h) and discharges (Q) along a single waterway or a network of interactive waterways. Complex hydraulic phenomena such as reverse flows, backwater effects from tributary inflows and hydraulic structures, flow accelerations, levee overtopping, etc. may be properly accounted for in dynamic routing. However, such phenomena are neglected by the simple hydrologic routing techniques, and yet these are sometimes selected for such applications because data for optimizing their parameters is more readily available than for dynamic routing.

Dynamic routing has been applied when there was a substantial amount of cross-sectional data available to characterize the cross-sectional area (A) and top width (B) as known functions of h. This required the existence of detailed hydrographic survey information and topographical maps, as well as considerable time consuming effort by

*Senior Res. Hydrologist & Res. Hydrologist, National Weather Service Hydrologic Res. Laboratory, 8060 13th St., Silver Spring, MD 20910.

hydrologists to reduce the basic cross-sectional information to the form of A and B as specified tabular functions of h. In order to eliminate the necessity for using such costly and often unavailable detailed data, a very powerful and computationally efficient parameter optimization methodology which utilizes minimal cross-sectional information was developed for application by the National Weather Service hydrologists for flood forecasting. Also, this methodology could be used advantageously by hydrologists/engineers concerned with unsteady flow prediction in waterways of many developing countries where detailed cross-sectional data is prohibitively expensive to obtain because of the remote locations of the waterways and the relative magnitude of the effort required for a single engineering study.

Theory and Background

The Saint-Venant equations of unsteady flow consist of a conservation of mass equation, i.e.

$$\partial Q/\partial x + \partial(A + A_o)/\partial t - q = 0 \tag{1}$$

and a conservation of momentum equation, i.e.,

$$\partial Q/\partial t + \partial(Q^2/A)/\partial x + gA(\partial h/\partial x + S_f) - qv_x = 0 \tag{2}$$

where:

$$S_f = n^2 \; |Q|Q/(2.21 \; A^2 \; R^{4/3}) \tag{3}$$

in which x is the distance along the longitudinal axis of the waterway, t is time, Q is discharge, A is active cross-sectional area, A_o is inactive (off-channel storage) area, q is lateral inflow (positive) or outflow (negative), g is the gravity acceleration constant, h is water surface elevation or stage, v_x is the velocity of the lateral flow in the x-direction, S_f is the friction slope computed by Manning's equation, n is the Manning coefficient, and R is the hydraulic radius.

In this paper, a dynamic routing model, FLDWAV, based on Eqs (1) and (2) which are solved by an implicit 4-point nonlinear finite difference technique described elsewhere (Fread, 1985; Fread and Smith, 1978) is used to implement and test the optimization methodology.

Data normally required to calibrate a dynamic routing model are: 1) cross-sectional area (A) and width (B) as a function of water surface elevation (h) for sections representative of the routing reach, 2) the Manning n which may vary with either elevation or discharge throughout the routing reach, 3) observed discharge and stage hydrographs at the upstream end of the routing reach, and either a stage or discharge hydrograph at the downstream extremity of the routing reach.

To avoid the costly and time-consuming tasks of gathering detailed cross-section data and then reducing the data into tables of top width (B) and water elevation (h), simple approximations are used to represent an average cross-section within each routing reach. A power function, $B_c = k_c Y_c^{m_c}$, is used for the channel and another power function, $B_f = k_f Y_f^{m_f}$, is used for the floodplain. The parameters k_c, m_c, k_f,m_f are estimated from 1) topographical maps, 2) visual inspection of a few easily accessible cross-sections, and/or 3) a few available cross-sections of the river. The shape parameter (m_c) can be easily computed, i.e.,

$$m_c = (\log B_2 - \log B_1)/(\log Y_2 - \log Y_1) \qquad (4)$$

in which B_2 and Y_2 are the estimated bank-full width and depth and B_1 and Y_1 are estimates of an intermediate width and depth. The scaling parameter (k_c) is computed from the basic power function, i.e., $k_c = B/Y^{m_c}$. Similarly the shape and scaling parameters for the floodplain can be computed from estimates of the floodplain widths and depths. Sometimes it is appropriate simply to estimate the shape parameter, i.e., rectangular-shape (m=0), parabolic shape (m=0.5), triangular shape (m=1.0) or -shapes (m>1). The parameter optimization methodology which has been programmed as an integral option within the FLDWAV routing model allows the k and m parameters to be specified directly or to be computed by the program from the specified B and Y values for each routing reach.

When unusual cross-sections exist in a routing reach, e.g., at a bridge, dam, or some natural constriction, those cross-sections' top width and elevation tables may be specified; they each remain distinct from the average section described by the two power functions.

A parameter optimization algorithm within FLDWAV iteratively determines the best value for the Manning n which is allowed to vary with h or Q for each reach of waterway bounded by water level recorders. An objective function defined as the difference between the computed and observed upstream stage hydrographs for several ranges of flow is minimized by a Newton-Raphson technique (Fread and Smith, 1978; Fread, 1985). A numerical derivative is used in lieu of the analytical derivative for the rate of change of the objective function with respect to the change in the Manning n. With starting values for n based on an assumption of steady flow or simply using a reasonable estimate, convergence to an optimal set of values is obtained in three to four iterations, i.e., the optimal n relation with h or Q for the reach of water bounded by known stage hydrographs can be obtained within three to four evaluations of the objective function; an evaluation consists of routing the flood hydrograph through the reach and comparing computed and observed upstream stage values. An option in FLDWAV allows the hydrologist to estimate a range of minimum and maximum n values within which the optimal n values must reside. When the optimal values are outside the specified min-max range, the cross section is automatically reduced or increased sufficiently to allow the next optimization to yield n values within the allowable range.

The optimization algorithm can be applied to multiple routing reaches, commencing with the most upstream reach and progressing reach-by-reach in the downstream direction. An observed discharge hydrograph is used as an upstream boundary condition for the most upstream reach. Then, the discharges computed at the downstream boundary using the optimal n values are stored internally by the program and used as the upstream boundary condition for the next downstream reach. Dendritic river systems are automatically decomposed into a series of multiple-reach rivers. Tributaries are optimized before the main-stem river and their flows are added to the main stem as lateral inflow(s).

This parameter optimization methodology offers several advantages over one described by Wormleaton and Karmegam (1984) which required 26 iterations to find the optimal parameters. It was restricted to a trapezoidal shaped cross-section with a constant n value and was for a single routing reach bounded by gaging stations with observed stage and discharge hydrographs from a previous flood.

The optimal n values obtained via the parameter optimization methodology presented herein is limited to expected applications where the maximum discharges do not greatly exceed those used in the optimization. Also, the methodology is best suited for applications where flood predictions are primarily required at locations along the waterway where level recorders exist. Unless detailed cross-sectional information at significant constriction or expansions is utilized in the optimization methodology, the cross sections throughout each routing reach should be generally uniform.

Applications

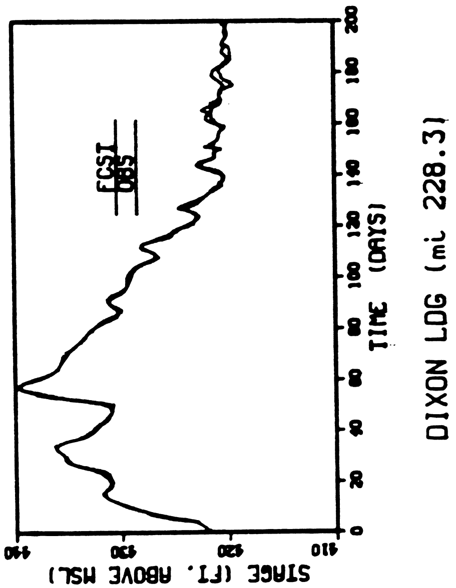
The new parameter optimization methodology has been tested on the following four river systems:

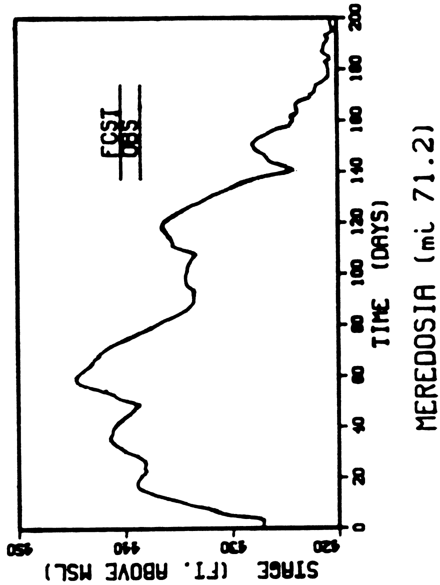
Lower Mississippi. This is a 292 mi (470 km) reach of the Lower Mississippi River consisting of eight water level recorders. The discharge is known at the most upstream station. This reach of the Lower Mississippi is contained within levees for most of its length. The average channel slope is an extremely mild 0.0000064. The discharge varies from low flows of about 100,000 cfs (2,832 m/s) to flood discharges of over 1,200,000 cfs (33,985 m/s). A total of 25 cross sections located at unequal intervals including the locations of the level recorders were used in the computations. The results of the parameter optimization are shown in Table 1. The effectiveness of the optimization is represented by the root-mean-square (rms) error between the computed and observed stage hydrographs at each level recorder. The average rms value is 0.37 ft (0.11 m). Also, this is presented as a percentage of the total change in water elevation which is only 2.4 percent. The average number of iterations or times that the flood is routed through each reach is three. Although the n values vary with discharge, the average value is shown for each reach.

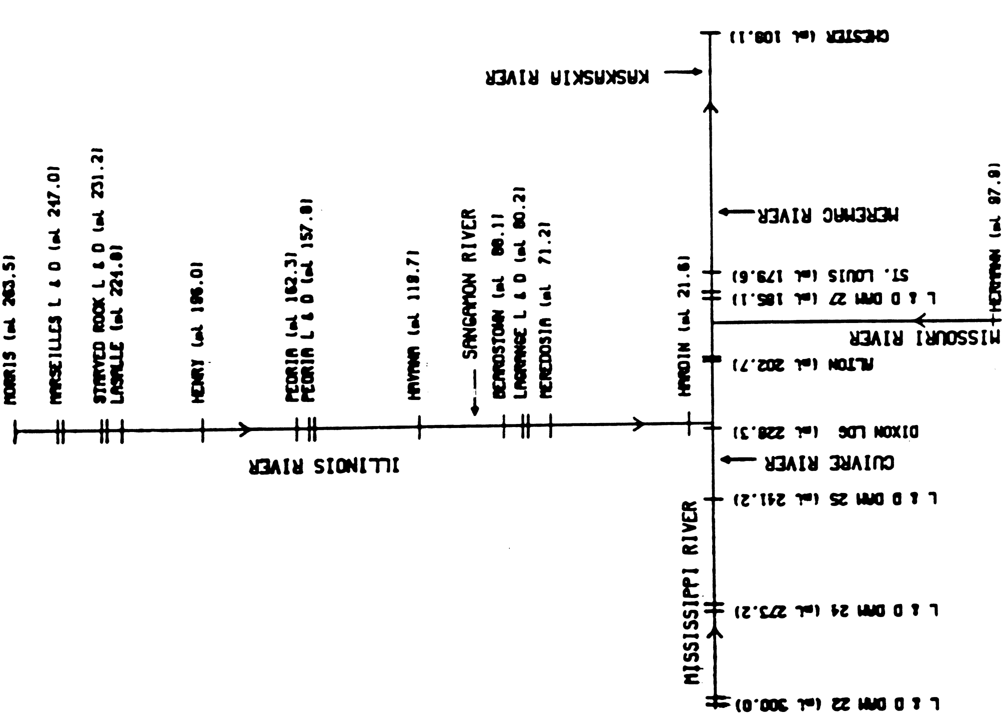
Ohio-Mississippi. This is a dendritic river system consisting of 393 miles (633 km) of the Mississippi, Ohio, Cumberland, and Tennessee Rivers with a total of 16 water level recorders and discharge measurements at the most upstream stations on each of the four rivers. The channel bottom slope is mild, varying from about 0.000047 to 0.000095. Each branch of the river system is influenced by backwater from downstream branches. Total discharge through the system varies from about 120,000 cfs (3,398 cms) to flood flows of 1,700,000 cfs (48,145 cms). A total of 45 cross-sections located at unequal intervals were used in the computations. The results of the parameter optimization are shown in Table 1. The average rms error is 0.62 ft. (0.19 m) representing

Table 1. Optimization Results for Four Major River Systems

RIVER SYSTEM River	Station	Root-Mean-Square Error ft (m)	Percent	Iter- ations	n_{avg}
L. MISSISSIPPI					
L. Mississippi	Red River Ldg.	0.11 (.03)	0.5	3	.025
-	Baton Rouge	0.16 (.05)	0.7	3	.018
-	Donaldsonville	0.16 (.05)	0.9	3	.015
-	Carrollton	0.27 (.08)	2.5	2	.014
-	Pt. Hache	0.27 (.08)	4.7	3	.015
		Avg 0.19 (.06)	Avg 1.9	Avg 3	
OHIO-MISSISSIPPI					
Ohio	Shawneetown	0.65 (.20)	2.3	3	.018
-	Golconda	0.67 (.20)	2.4	4	.021
-	Paducah	0.68 (.21)	2.7	3	.023
-	Metropolis	0.59 (.18)	2.8	4	.020
-	Cairo	1.04 (.32)	4.8	6	.026
Cumberland	Berkley Dam T.W.	0.77 (.23)	2.8	4	.016
Tennessee	Kentucky Dam T.W.	0.85 (.26)	3.4	4	.036
U. Mississippi	Chester	0.34 (.10)	1.4	3	.026
-	Grand Tower	0.47 (.14)	2.0	3	.032
-	Cape Girardeau	0.51 (.15)	2.2	2	.024
L. Mississippi	New Madrid	0.72 (.22)	3.8	3	.022
		0.66 (.20)	2.8	3	
ILLINOIS-MISSISSIPPI-MISSOURI					
Illinois	Morris	0.30 (.09)	2.2	2	.025
-	La Salle	0.21 (.06)	1.2	4	.030
-	Henry	0.30 (.09)	2.8	4	.020
-	Peoria	0.21 (.06)	1.6	5	.046
-	Havana	0.63 (.19)	3.3	3	.038
-	Beardstown	0.50 (.15)	2.7	3	.033
-	Meredosia	0.15 (.05)	0.6	3	.022
-	Harding	0.54 (.16)	2.8	3	.017
Mississippi	L&D No. 24 T.W.	0.38 (.12)	1.8	3	.028
-	L&D No. 25 T.W.	0.41 (.13)	1.8	2	.039
-	Dixon Ldg.	0.35 (.11)	1.7	4	.033
-	Alton	0.18 (.05)	0.6	3	.023
-	St. Louis	0.47 (.14)	1.1	4	.024
Missouri	Hermann	0.68 (.21)	2.7	4	.034
		0.38 (.12)	1.9	3	
COLUMBIA					
Columbia	Bonneville	0.22 (.07)	2.4	4	.040
-	Washougal	0.27 (.08)	4.5	3	.032
-	Vancouver	0.31 (.09)	6.2	4	.024
-	Columbia	0.40 (.12)	8.3	4	.016
-	Wauna	0.74 (.23)	11.4	3	.022
Willamette	Portland	0.30 (.09)	5.0	3	.024
		0.37 (.11)	6.3	4	


DIXON LDG (mi 228.3)
TIME (DAYS)
STAGE (FT. ABOVE MSL)
FCST
OBS


MEREDOSIA (mi 71.2)
TIME (DAYS)
STAGE (FT. ABOVE MSL)
FCST
OBS


ILLINOIS RIVER
MISSISSIPPI RIVER
MISSOURI RIVER
SANGAMON RIVER
KASKASKIA RIVER
MEREMAC RIVER
CUIVRE RIVER

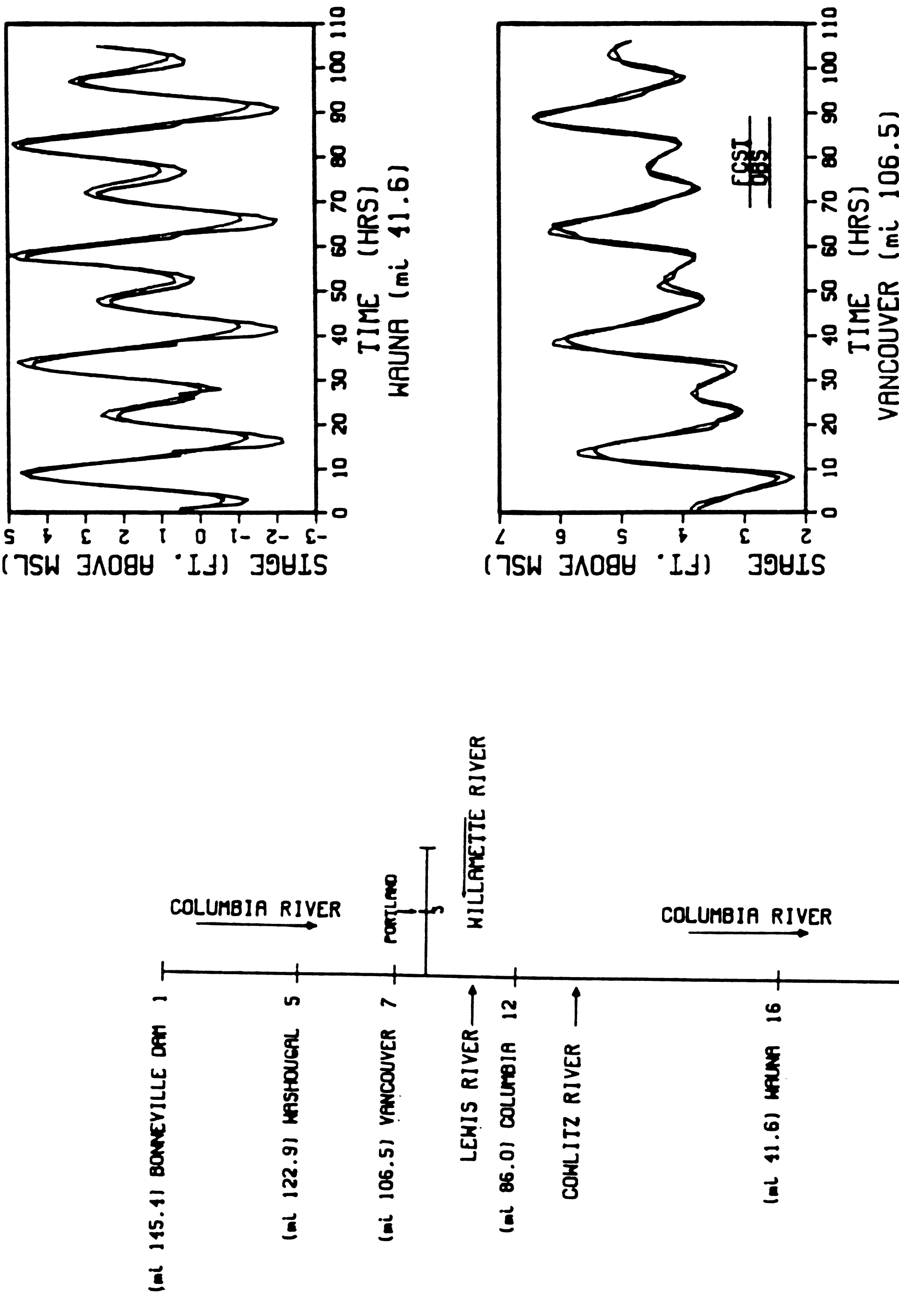


Fig. 2 -- Columbia River System

2.5 percent of the change in water elevation during the flood. An average of only three iterations were required.

Illinois-Mississippi-Missouri. A schematic of this dendritic river system is shown in Figure 1. It consists of 463 miles (741 km) of the Mississippi, Illinois, and Missouri Rivers with a total of 9 lock and dams and 18 level recorders. Discharges are known at the upstream end of each river. The channel bottom slope is mild, varying from 0.00002 to 0.0002. The total discharge through the system varied from 3,600 cfs (102 cms) to 885,000 cfs (25,064 cms). A total of 117 cross sections located at unequal intervals were used in the computations. Results of the parameter optimization are shown in Table 1. The average rms error was 0.38 ft (0.12 m) representing 1.9 percent of the change in water elevation during the flood. About three iterations were required for each reach. Comparisons of computed and observed water elevation at Meredosia and Dixon Landing are shown in Figure 1.

Columbia-Willamette. The lower 128 miles (206 km) of the Columbia River and the lower 24.4 mile (32 km) reach of the Willamette River have a very flat bottom slope (0.000011), and the flows are quite affected by the tide from the Pacific Ocean. A schematic of the reaches modeled are shown in Figure 2. The tidal effect extends as far upstream as the tailwater of Bonneville Dam during periods of low flow. Reverse flows can occur as far upstream as Vancouver. A total of 25 cross sections located at unequal distance intervals were used in the computations. Results of parameter optimization for a 3-day low flow period are shown in Table 1. The average rms error was 0.37 ft. (0.11 m) or 6.9 percent of the change in water elevation. An average of four iterations were required. Comparisons of computed and observed water elevations at Wauna and Vancouver are shown in Figure 2.

Summary

A parameter optimization methodology for dynamic flood routing models has been developed which for some applications can eliminate the need for costly and time-consuming detailed cross-sectional information. The cross sections within a reach are approximated by separate power functions for the channel and the floodplain. A very efficient optimization algorithm determines the optimal Manning n values which may vary with either water elevation or discharge. Minimum-maximum constraints can be imposed on the optimal n values. One or more cross sections within a reach may be specified in detail by a table of top widths and elevations. The methodology has been tested with promising results on several large river systems; the average rms error was 0.44 ft (0.13 m) or 2.9 percent of the change in stage.

Appendix - References

Fread, D.L., "Channel Routing," Chapter 14, Hydrological Forecasting, (Ed: M.G. Anderson and T.P. Burt) John Wiley, 1985, pp. 437-503.

Fread, D.L. and Smith, G.F., "Calibration Technique for 1-D Unsteady Flow Models," ASCE, 104(7), Paper 13892, Jul 1978, pp. 1027-1044.

Wormleaton, P.R. and Karmegam, M.,"Parameter Optimization in Flood Routing," ASCE, 110(12), Paper 19352, Dec. 1984, pp. 1799-1814.